

Energy Loss by Gravitational Viscosity

Ernst Fischer*

e.fischer.stolberg@t-online.de

Abstract

Due to Lorentz invariance of General Relativity gravitational interaction is limited to the speed of light. Thus for particles, moving within a matter field, retardation leads to loss of energy by emission of gravitational radiation. This 'gravitomagnetic' effect, applied to motion in homogeneous mass filled space, acts like a viscous force, slowing down every motion in the universe on the Hubble time scale. The energy loss rate exactly equals the red shift of photons in an expanding universe, thus showing the equivalence of wavelength stretching in the wave picture and energy loss in the photon picture. The loss mechanism is not restricted to an expanding universe, however, but would also be present in a static Einstein universe.

1 Introduction

Today the theory of general relativity (GRT) is accepted as the correct description of gravitation, but due to its non-linear character and the complicated mathematical formalism practical applications have been rather limited and, wherever it appears acceptable, the much simpler formalism of Newtonian theory is used, which appears as a good approximation in the limit of weak fields and low velocities. One of the most important differences from Newtonian gravity is the Lorentz invariance of GRT, following from the requirement that gravitational interaction should be independent from any preferred reference system other than the complete universe.

One consequence of this requirement is the fact that gravitational interaction is not instantaneous but limited to the speed of light, leading to similar radiation effects as we know them from electromagnetic interaction. Changes of the matter or energy distribution lead to changes of the metric, expanding into space by the speed of light. Though this effect has been already discussed by Einstein in his famous quadrupole formula, it remained of little practical interest up to the last two decades. The first observation, which could be attributed to this gravitational radiation, was the detection that the frequency of double pulsars was slowing down exactly according to GRT (Weisberg and Taylor (1984) [6]).

*Auf der Höhe 82, D-52223 Stolberg, Germany

Besides that, several attempts have been made to detect gravitational waves directly, which should be emitted by cosmic catastrophes such as collisions of black holes or other huge matter concentrations. An overview of the theoretical models developed to describe such events has been given by Poisson (2004) [4].

But while these activities concentrate on processes, which produce very strong deformations of space-time, there should be also effects in the weak field limit, which cannot be detected directly, but which may be important, when they accumulate over cosmic distances or times. They may influence the development of cosmic structures like the formation of galaxies or clusters.

The primary effect, which makes up the difference between Newtonian theory and GRT is the finite velocity of gravitational interaction. Thus we will try to investigate such processes in the limit of Newtonian physics, but with the modification that the interaction velocity is given by the speed of light, just as we are accustomed from the theory of electromagnetic interaction. In analogy to the magnetic effects observed in the interaction of moving electrical charges, these deviations from Newtonian theory are mostly called 'gravitomagnetic' and their existence has been proved with high accuracy in lunar and planetary ranging observations (see e.g. Nordtvedt (2003) [3]).

In electromagnetic theory of moving charges static potentials have to be replaced by the retarded Liénard-Wiechert potentials. That this retardation has an effect on the motion of particles also in gravitational theory can be immediately recognised from a simple thought experiment. We consider a particle moving on a line between two equal masses. When the particle at time t is just at equal distance r from both masses, the gravitational force at this moment is determined by the distance at the time $t - r/c$. That means that the distance to the mass in direction of the motion is increased and the distance to the other mass is reduced. Thus the particle feels a force, which is directed opposite to the direction of motion, thus reducing its momentum and energy of motion.

Of course, if we try to sum up the forces exerted on a test particle by the total matter in a Euclidean universe, we are confronted with the infinity problems, well known from the Olbers paradox. But also here general relativity supplies us with a remedy, or better to say, with two possible solutions. One is the assumption, favoured by main stream physics of today, that the universe is expanding and thus, even if the size of the universe is infinite, interaction is limited to that fraction of matter, which can be causally connected to the test particle. But possible is also the other explanation, originally proposed by Einstein, the assumption that space is curved and thus of finite size and matter content. In the sequel we will consider both possibilities and show that they lead to similar results.

2 Energy loss in expanding space

To demonstrate, how the finite speed of gravitational interaction affects the energy balance of moving particles, as a toy model we consider the motion of a test mass in a universe, expanding at a constant rate, but keeping the density

constant by some creation process similar to that proposed by Hoyle et al. (1993) [2]. This does not mean that this model appears more attractive than the presently favoured 'concordance model'. But it allows a mathematically very simple description of the effects, as the relevant quantities, matter density and expansion velocity, do not depend on time.

Starting with Newtonian physics, the gravitational potential at some point generated by masses m_i at distances r_i is given by

$$U = \sum_i \frac{Gm_i}{r_i} \quad (1)$$

(G is the gravitational constant). But if gravitational interaction is limited by the speed of light, similar to electromagnetic interaction, if the position of the masses changes with time, we have to use retarded potentials analog to the Liénard-Wiechert potentials. Instead of the distance at the local time t , we have to insert the distance at time $t - \tau$, where τ is the running time of the signal $\tau = r_i/c$.

$$U^*(t) = \sum_i \frac{Gm_i}{r_i(t - \tau)}. \quad (2)$$

In the static case this does not change the result, but for a moving particle the retardation parameter $t - \tau$ changes with time. As a result there occurs an additional gradient of the potential, leading to a retarding force on the particle.

We should stress here that analogous to electromagnetic interaction retardation does not lead to aberration effects, as Lorentz invariance requires that there exists no preferred reference frame. Thus interaction can depend only on the scalar retarded potential, not on the direction of forces as determined from Euclidean geometry. A detailed discussion of this fact has been given by Carlip (2000) [1]. While a test particle, linearly moving in the field of homogeneously distributed masses, feels a retarding force, as the distance to the individual masses changes with time, in circular motion of an isolated two-body system there is no retarding force. As the distance of the masses does not change, angular momentum is conserved. Thus in the solar system deviations from Newtonian physics occur only as tiny corrections of higher order in v/c due to rotation, tidal effects and eccentricity of the moving bodies. In linear motion there exists an effect of first order in v/c , however.

Let us assume a particle moving at velocity v in positive x-direction of Euclidean space. Using a comoving coordinate system, we can regard all the other masses as moving with respect to it with speed $-v$. Thus their distance changes with time by

$$\frac{dr_i}{dt} = \frac{dr_i}{dx} \cdot \frac{dx}{dt} = \frac{x_i}{r_i} \cdot \frac{dx}{dt} = -v \frac{x_i}{r_i} \quad (3)$$

In the comoving system there is a gradient of the retarded potential, which is felt by the particle as a retarding force. It can be expressed by a power series

$$\frac{dU^*}{dx} = \sum_i Gm_i \frac{d}{dx} \sum_{n=0}^{\infty} \frac{d^n}{d\tau^n} \left(\frac{1}{r_i} \right) \frac{\tau^n}{n!}. \quad (4)$$

Restricting to the linear term of the series expansion with $\tau = r_i/c$ we get

$$\begin{aligned}\frac{dU^*}{dx} &= \sum_i Gm_i \frac{d}{dx} \left(\frac{1}{r_i} + \frac{vx_i}{r_i^3} \frac{r_i}{c} \right) \\ &= \sum_i Gm_i \left[-\frac{x_i}{r_i^3} + \frac{v}{c} \left(\frac{1}{r_i^2} - \frac{2x_i^2}{r_i^4} \right) \right]\end{aligned}\quad (5)$$

We can now apply this formula to the motion of particles in an homogeneous matter filled space of density ϱ . In this case the mass in a toroidal volume element at distance r_i is $m_i = \varrho dV_i = 2\pi\varrho r_i^2 \cos\vartheta d\vartheta dr$, where the projected distance in the direction of motion is expressed by $x = r \sin\vartheta$. Then integration of the potential gradient induced by the matter is

$$\frac{dU^*}{dx} = 2\pi G\varrho \int_0^{r_{max}} \int_{-\pi/2}^{\pi/2} Y(r, \vartheta) r^2 \cos\vartheta d\vartheta dr \quad (6)$$

$$\text{with } Y(r, \vartheta) = \left[\frac{\sin\vartheta}{r^2} + \frac{v}{c} \left(\frac{1}{r^2} - \frac{2\sin^2\vartheta}{r^2} \right) \right] \quad (7)$$

The quantity r_{max} has been introduced to express that integration has to be limited to the causal sphere, that means, to the volume range, which can interact, when this interaction is limited to the speed of light. Thus, if space is expanding at a rate given by the Hubble constant H , the limit is $r_{max} = c/H$.

From symmetry considerations it is immediately clear that the first term of the integral is zero, but the second term gives a non-vanishing contribution proportional to v/c :

$$\frac{dU^*}{dx} = 2\pi G\varrho r_{max} \cdot \frac{v}{c} \left(\sin\vartheta - \frac{2}{3} \sin^3\vartheta \right)_{-\pi/2}^{\pi/2} \quad (8)$$

corresponding to a force on a particle of mass m

$$F = -m \frac{dU^*}{dx} = -\frac{4\pi}{3} G\varrho r_{max} \cdot \frac{mv}{c} \quad (9)$$

Introducing $r_{max} = c/H$ there is a loss of momentum p proportional to the actual momentum

$$\frac{dp}{dt} = -\frac{4\pi}{3} \frac{G\varrho}{H} \cdot p, \quad (10)$$

and correspondingly the loss of kinetic energy is

$$\frac{dE}{dt} = \frac{d}{dt} \left(\frac{p^2}{2m} \right) = -\frac{8\pi}{3} \frac{G\varrho}{H} \cdot E. \quad (11)$$

As in an expanding universe the case of zero curvature corresponds to the critical density $\varrho_{cr} = 3H^2/(8\pi G)$, this energy loss corresponds exactly to that of photons $dE/dt = -HE$. That means that every moving particle in a mass filled

universe of critical density experiences the same energy loss as we ascribe it to photons and explain it by stretching of wavelength by the expansion of space. Or looking at it the other way round: What we explain by stretching of wavelengths in a pure wavelike picture of electromagnetic radiation, may as well be considered as part of a general energy loss mechanism of all matter, if we regard photons as individual entities, which have inertia $h\nu/c^2$ and feel gravity in the same way as massive particles.

The situation is much the same as in the experiments by Pound and Rebka (1960) [5] to measure gravitational red shift. The results can be explained in a geometrical way by time dilatation, caused by spacetime deformation by the surrounding matter, but as well by the difference of the gravitational potential between the points of emission and detection of radiation. Equivalence of both pictures results from conservation of energy, which is valid, when potential energy is included.

Of course, that extending the quasi-Newtonian approximation to photons is allowed, has not been proved. But the fact that the numerical values are exactly equal and that energy conservation should be valid, independent of the nature of the moving mass or quantum, is an indication that the energy loss effect is a common phenomenon to massive and massless particles. Otherwise the principle of equivalence, which is a corner stone of GRT, would be violated. Mass filled space appears as a kind of viscous medium, in which energy of all moving particles is dissipated to the gravitational potential.

3 Energy loss in curved space

In the last section we have shown that the cosmic energy loss mechanism must not necessarily be attributed to an expansion of space. The basic proposition is the finite speed of gravitational interaction, which is a consequence of the Lorentz invariance of the basic GRT equations. It is this Lorentz invariance, which requires that any homogeneous space solution of the basic equations must be either expanding or spatially curved. Energy loss by gravitational viscosity, as we have called it, must be present, too, if space is not expanding. We only have to find another explanation, why gravitational interaction is limited to a finite amount of matter, if this amount is not limited by causal connection.

There is of course such a possibility, when the total amount of matter in the universe is limited, as it was proposed in Einstein's first introduction of GRT. He proposed that the universe is static, but has a positive curvature, so that the total matter content is limited. We can easily adjust the derivation of cosmic energy loss to this case, we only have to change the size of the volume elements and the limits of integration.

We consider a homogeneous spherical universe with a positive radius of curvature R . As in Euclidean space we assume that gravity acts along the geodesic lines and that the strength of interaction decreases with the square of the length measured along these lines. Denoting this distance by $r = R \cdot \varphi$, the main difference compared to Euclidean space is the reduction of the volume element at

distance r by the factor $(\sin \varphi/\varphi)^2$. There exists, of course, no limitation to the length of the geodesic lines, so that the value of φ may extend to infinity. But due to the limited size of the volume elements the value of the integral remains finite. Thus in eq.(6) we only have to replace the integral

$$\int_0^{r_{max}} dr = r_{max} \quad \text{by} \quad R \int_0^\infty \frac{\sin^2 \varphi}{\varphi^2} d\varphi = \frac{R\pi}{2}, \quad (12)$$

leading to an energy loss

$$\frac{dE}{dt} = -\frac{4\pi^2}{3} \frac{G\varrho R}{c} \cdot E. \quad (13)$$

The relation between the radius of curvature and the density of matter in the static Einstein universe is given by $R = \sqrt{c^2/(4\pi G\varrho)}$. Thus, if we again identify the energy reduction factor in eq.(13) with the Hubble constant H , the density of matter is related to H by

$$\varrho = \frac{9H^2}{4\pi^3 G}, \quad (14)$$

which differs not much from the critical density in an expanding universe. The factor is $\varrho/\varrho_{cr} = 6/\pi^2$. From the viewpoint of red shift or global energy loss there is no reason, to prefer one model against the other.

4 Discussion

As has been shown in the last sections, the gravitomagnetic effects inherent to GRT or, as we call it in the context of global effects, 'gravitational viscosity' can explain observed red shift with and without any expansion of space as well. To prove, if the general energy loss mechanism on cosmic scale really exists, our possibilities are rather limited. To prove the existence, we would have to follow the path of moving particles over very long periods of time and to know the initial energy of motion exactly. But unfortunately the only particles, for which the initial energy is known from the emission process, are photons - at least if we assume that the laws of quantum electrodynamics have not changed with time.

Observations within the solar system have unambiguously shown, that the gravitomagnetic effects exist. The planetary and lunar ranging experiments (see [3]), by which the distance from earth to moon could be measured down to an accuracy limit of cm, have demonstrated the validity of GRT in an impressive way. The effect of global energy loss is just at the accuracy limit of these measurements. It would change the distance to the moon by about 3 cm per year. But there are so many perturbing effects of the same or higher magnitude within the solar system that the observed changes cannot be unambiguously attributed to the global loss mechanism. And even if we succeed to isolate the global change from all the other perturbations, it remains the question, if it is due to expansion or curvature.

What we need to decide this question, are measurements, which do not rely on red shift or similar energy loss, but only on geometric observations like angular size or mean number density of distant galaxies. But also the interpretation of these measurements is hampered by the possibility that the properties of galaxies may have changed with time. Thus a conclusive decision is still missing. But an increasing number of observations shows that the most distant galaxies look quite similar as those nearby, casting doubts on the assumption that a strong development in size or properties has taken place since the light has been emitted, which is reaching us now.

There is one indirect hint, however, that gravitational viscosity, at least by part, is responsible for the observed energy loss of moving matter and consequently also for red shift of photons. Without assuming some dissipation of kinetic energy, which matter acquires while contracting into galaxies or clusters, formation of these structures cannot be explained. Gravitational viscosity supplies this dissipation effect. The fact that we observe galaxies as spirals and not as circular disks, can easily be attributed to the loss of angular momentum during formation of structures on the Hubble time scale.

Thus, if we take GRT serious as the correct theory of gravitation, we should do it with all its consequences. We should not regard Doppler effect in an expanding universe as the only possible explanation of observed red shifts. Taking into account energy loss by gravitational viscosity, may help us to a better understanding of the formation phenomena of large scale cosmic structures and it may even question present days assumptions on the age of the universe, derived from observed red shift.

References

- [1] Carlip S., 2000, Physics Letters A, 267, 81-87
- [2] Hoyle F. et al., 1993, AJ 410, 437
- [3] Nordtvedt K., 2003, arXiv:gr-qc/0301024
- [4] Poisson E., 2004, *The motion of point particles in curved spacetime*, Living Rev. Relativity 7 and (arXiv gr-qc/0306052)
- [5] Pound R.V. and Rebka G.A. Jr., 1960, Phys. Rev. Lett. 4, 337
- [6] Weisberg I.M. and Taylor I.H., 1984, Phys. Rev. Lett. 52, 1348